1.

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval $[3.5, 4]$

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

a) $f(x) = \ln(2x-5) + 2x^2 - 30$

$f(3.5) = \ln(2\times3.5 - 5) + 2(3.5)^2 - 30 = -4.81$

$f(4) = 3.10$ (1) $\Rightarrow$ In the interval $[3.5, 4]$ we see a change in sign $\Rightarrow$ there is a root, α, in this interval. (1)

b) Newton - Raphson : $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$

$x_0 = 4$

$f(4) = 3.099$

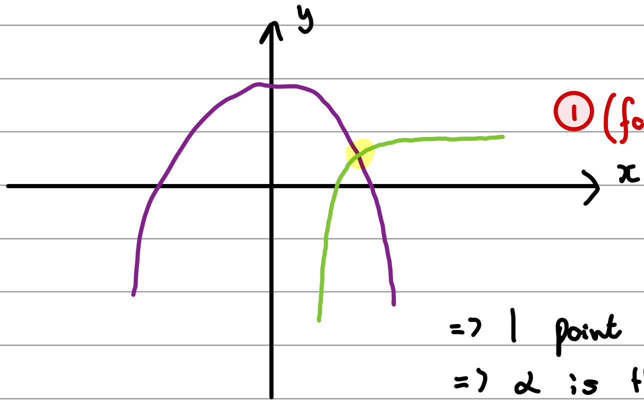
$f'(4) = 16.67$

$\Rightarrow x_1 = 4 - \dfrac{3.099}{16.67}$ (1) $= 3.81409\ldots \Rightarrow \underline{\underline{x_1 = 3.81}}$ (1)

c) $f(x) = 0 \Rightarrow \ln(2x-5) + 2x^2 - 30 = 0$

$\Rightarrow \ln(2x-5) = 30 - 2x^2$

$30 - 2x^2$



(1) (for sketch)

$\Rightarrow$ 1 point of intersection

$\Rightarrow \alpha$ $\underline{\underline{\text{is}}}$ the only root.

2. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

a) Newton-Raphson formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = 2x^3 + x^2 - 1$

$f'(x) = 6x^2 + 2x$ ✓

$$x_{n+1} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$$ ✓

$$= \frac{x_n(6x_n^2 + 2x_n)}{6x_n^2 + 2x_n} - \left(\frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}\right)$$

$$= \frac{x_n(6x_n^2 + 2x_n) - 2x_n^3 - x_n^2 + 1}{6x_n^2 + 2x_n}$$

$$= \frac{6x_n^3 + 2x_n^2 - 2x_n^3 - x_n^2 + 1}{6x_n^2 + 2x_n}$$

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$ ✓

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

b) $$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

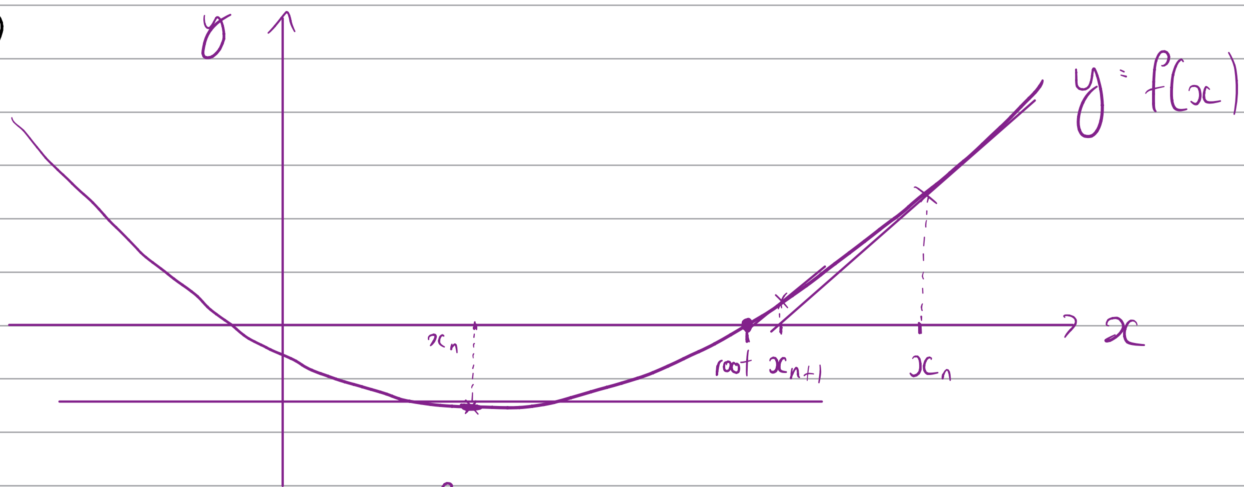
$$x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)} = \underline{\underline{\frac{3}{4}}}$$ ✓1

$$x_3 = \frac{4\left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^2 + 1}{6\left(\frac{3}{4}\right)^2 + 2\left(\frac{3}{4}\right)} = \underline{\underline{\frac{2}{3}}}$$ ✓1

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

c)



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

at $f'(x_n) = 0$, at a turning point, $\frac{f(x_n)}{f'(x_n)}$ is undefined.

For $f(x) = 2x^3 + x^2 - 1$, $f'(x) = 6x^2 + 2x$.

For $x_1 = 0$, $f'(0) = 6(0)^2 + 2(0) = 0$ $\therefore$ a turning point.

Consequently, the tangent will be horizontal, will not meet the x-axis and so not locate the root. ✓1